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Spanwise correlation of unsteady lift due to vortex shedding on the square cylinder: a time-frequency analysis

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SUMMARY:
In this work, Hilbert transform and Wavelet transform as time-frequency analysis tools were applied to the sectional lifts acquired from the central span of a square cylinder with an aspect ratio of 20.8. The generalized Morse wavelet, with its parameters optimized for better temporal resolution, was considered as the mother wavelet, and the synchrosqueezing technique was used for the post-processing. Hilbert transform and Wavelet transform mutually confirm the vortex shedding frequency fluctuating in real-time. As an example, at $Re = 3.9 \times 10^4$, results from Wavelet transform suggest a standard deviation of the instantaneous shedding frequency of at least 4% about the conventional Strouhal frequency (the time-averaged one). The extracted instantaneous shedding frequency is stochastic. The concerning properties such as probability density function, power spectra, and spanwise coherence have been investigated. The shown characteristics might easily recall those of turbulence. Finally, the spanwise correlation of unsteady lift can be reconstructed via the extracted instantaneous vortex shedding frequency.

Keywords: Vortex shedding, Square cylinder, Time-frequency analysis

1. INTRODUCTION

The spanwise correlation of unsteady lift due to vortex shedding is particularly of interest as it is relevant to the vortex induced vibration (VIV) and the combined VIV-galloping interaction (Chen et al., 2022; Mannini et al., 2014). Although reliably predicting this correlation remains very difficult as early noted by Bearman (1997), the work of Szepessy (1994) for a circular cylinder at a Reynolds number $Re = 4.3 \times 10^4$ has provided inspirational perspectives. Instantaneously, substantial variations were found in the phase of vortex shedding, and the phase drift between two spanwise cross sections shows random characteristics to some extent following the Gaussian probability density function. The presented work is inspired so but focuses on the possible variation of vortex shedding frequency in real time (as early conjectured by Szepessy (1994)), which could be more feasible in the future to incorporate into the various mathematical models for VIV.

2. EXPERIMENTS

Wind tunnel static tests were carried out in the boundary layer wind tunnel at the Institute of Steel Structures, Technische Universität Braunschweig, Germany. Fig. 1 shows the schematic of the square cylinder model. The blockage ratio led by this cylinder is 5%, defined as the ratio of the cylinder height d = 60 mm to the wind tunnel height 1200 mm. The aspect ratio of this cylinder model is about 20.8 between the two end-plates. Pressure measurement was performed with a 32-channel ESP mini-transducer implemented with the DTC Initium System to gain the sectional lift. The physical sampling frequency was found to fluctuate around 652 Hz (per second), digitally re-sampling of the raw data to a fixed 652 Hz was carried out. Experiments are mainly carried out at $Re = 3.9 \times 10^4$, at which the free-stream turbulence intensity is about 0.8%. It has been checked that the central $7d$ mid-span features the best uniformity. Results are presented for this range.

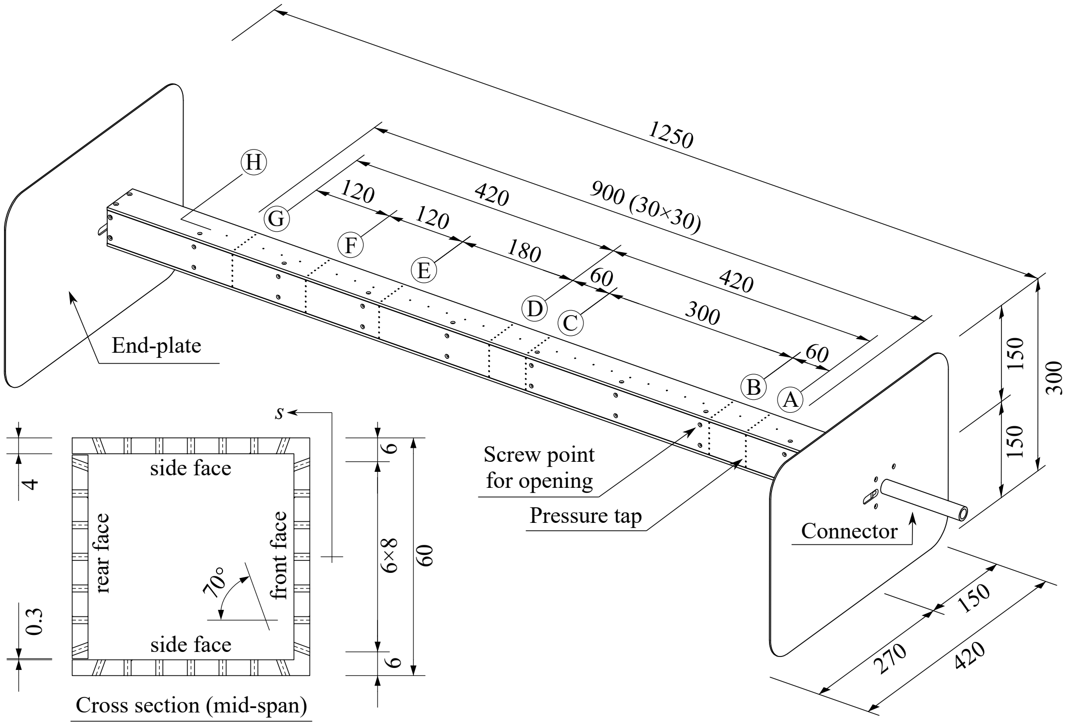


Figure 1. Schematic of the square cylinder wind tunnel model made of aluminum.

3. TIME-FREQUENCY TECHNIQUES

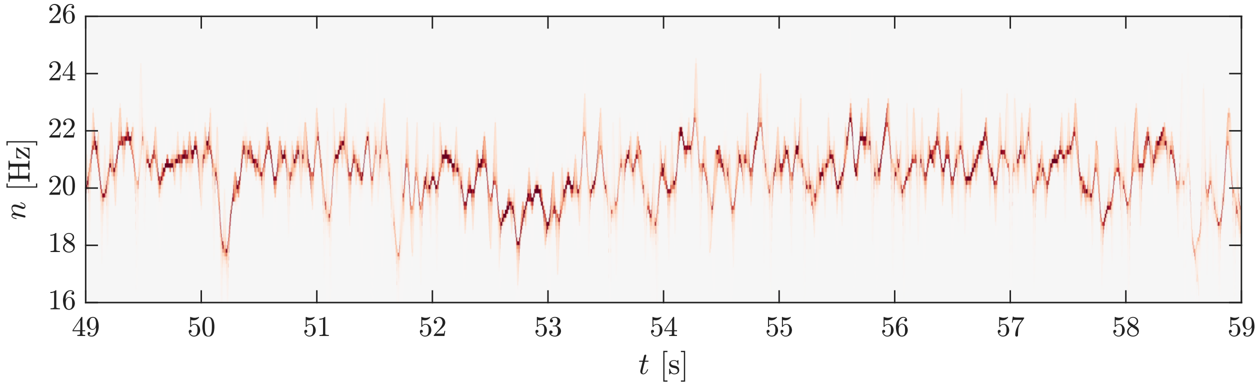
Two time-frequency techniques are used: Hilbert transform (HT) and Wavelet transform (WT). For WT, the generalized Morse wavelet (Lilly and Olhede, 2012) serves as the mother wavelet:

$$\hat{\psi}(\omega) = H(\omega) A_{\beta,\gamma} \omega^{\beta} e^{-\omega^{\gamma}} \tag{1}$$

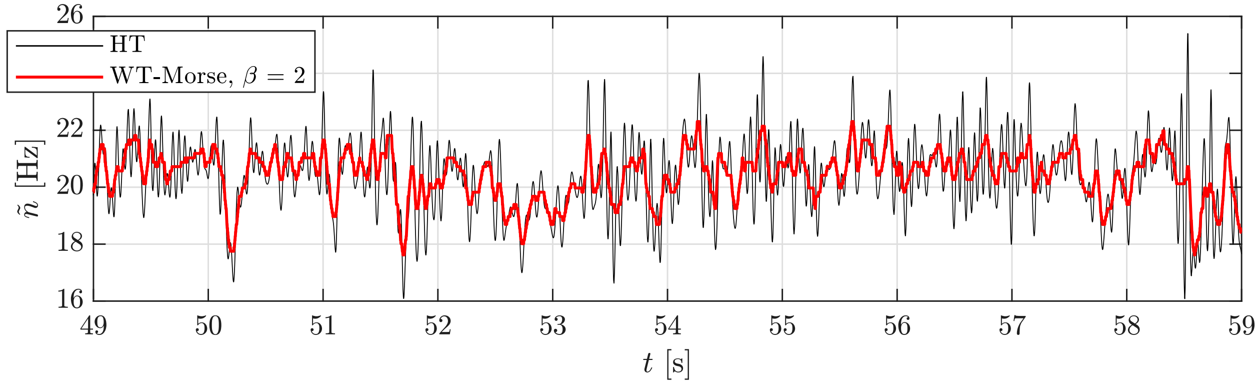
It is defined in the frequency domain (ω, circular frequency) and strictly analytical. $H(\omega)$ is the unit step function, $A_{\beta,\gamma}$ the normalization constant, β and γ two parameters to shape the wavelet. $\gamma = 3$ was set for a minimum Heisenberg area, and $\beta = 2$ was used for a better temporal resolution at the cost of frequency resolution. The Morse wavelet peaks at $\omega_p = (\beta/\gamma)^{1/\gamma}$, and $A_{\beta,\gamma}$ is so set that $\hat{\psi}(\omega_p) = 1$. The synchrosqueezing technique (SST) (Daubechies et al., 2011) was used for post-processing, transferring the time-scale representation of WT to the time-frequency representation. In this process, the smearing effect in the original spectrogram of WT was significantly reduced, leading to a sharpened time-frequency plot. The instantaneous frequency ($\tilde{n}$) is then determined at the ridge of the time-frequency representation, and extracted by the function *tfridge* of Matlab®. The raw data of sectional lift was first filtered with a zero-phase low-pass filter cutting-off at $2n_{st}$, being n_{st} the conventional Strouhal frequency about 20 Hz. The filtered signal is strongly single-ridge featured thus empirical mode decomposition is unnecessary for HT to extract $\tilde{n}$. Finally, the filtering process affects nearly not the spanwise correlation of unsteady lift (<0.4%).

4. RESULTS

Fig 2(a) shows the time-frequency spectrogram for the sectional lift (WT+SST), at the central cross section D. The concerning time window was purposely selected, within which it contains about 200 vortex shedding cycles and no strong amplitude modulation. Real-time fluctuation of the shedding frequency can be directly viewed. Fig 2(b) provides a comparison for the extracted $\tilde{n}$ by WT and by HT. Good agreement can be found in the low-frequency fluctuation part. The results of HT are more fluctuating, since HT is more capable to capture the subtle change in $\tilde{n}$ (thus fluctuation up to higher frequency) but also influenced more by the noise. For the relative standard deviation std$(\tilde{n})/n_{st}$, WT reports 4.2% and HT 6.35%. A not-bad similarity of $\tilde{n}$ to the Gaussian probability density function was found. For instance, skewness and kurtosis of $\tilde{n}$ by HT report -0.19 and 3.32.



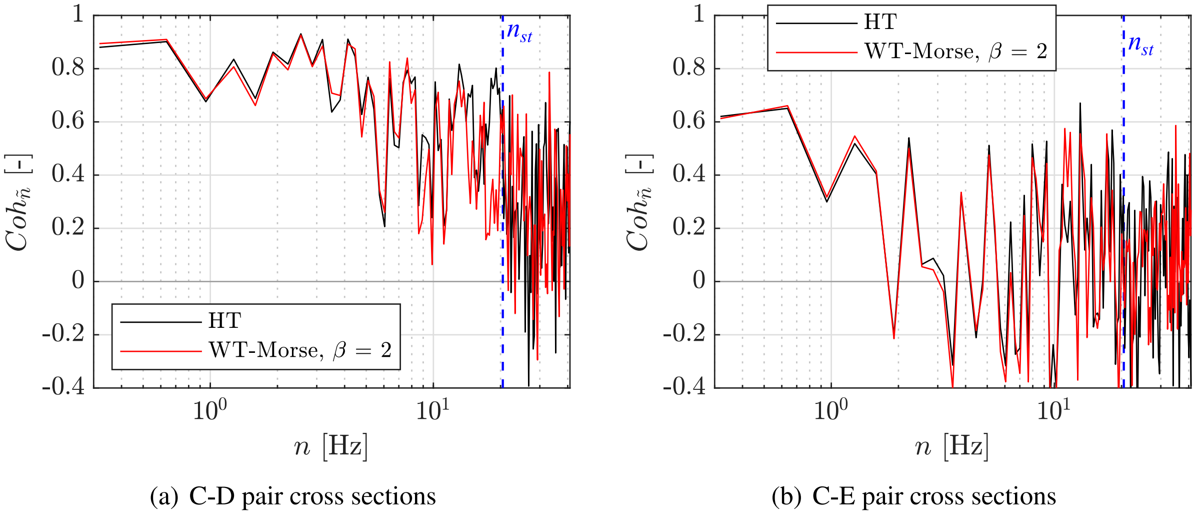
(a) Time-frequency spectrogram of WT, post-processed with SST. Color represents energy intensity.



(b) Comparision of the intantanenous frequency extracted by HT and WT (energy ridge of (a)).

Figure 2. The variation of vortex shedding frequency in real time.

The $\tilde{n}$ simultaneously extracted at two cross sections are found correlated. In the frequency domain, Fig. 3 shows the coherence (*Coh*) of $\tilde{n}$ for the C-D cross sections of $1d$ separation and C-E of $4d$. $Coh_{\tilde{n}}$ in general decreases with n, indicating that the low-frequency fluctuation in $\tilde{n}$ dominates the correlation. In view of Fig. 3, one may easily recall the similar characteristics of turbulence. Due to limited space, it is only to add that the spanwise correlation of unsteady lift has been confirmed reconstructable via $\tilde{n}$. Moreover, it was found that it is the low-frequency fluctuation portion in the instantaneous frequency (about $n < 1/2n_{st}$) playing the major role.



(a) C-D pair cross sections

(b) C-E pair cross sections

Figure 3. Coherence of the extracted $\tilde{n}$. Smoothing technique is used (every 2048 points and 80% overlapping).

5. CONCLUSIONS

At high Re, the vortex shedding process is instantaneously characterized by a fluctuating shedding frequency, mutually confirmed by two different time-frequency techniques. This fact has been early conjectured by Szepessy (1994) for the circular cylinder. For the square cylinder (at the investigated $Re = 3.9 \times 10^4$), the standard deviation of the fluctuating shedding frequency is at least 4% about the conventional Strouhal frequency. The extracted instantaneous frequency is stochastic, showing a not bad similarity to the Gaussian distribution for its probability density function. The instantaneous frequencies simultaneously extracted at two cross sections (of a spanwise separation) show meaningful coherence (in the frequency domain), whose shape is quite similar to that of turbulence and get decreased with the increase of spanwise separation. The spanwise correlation of unsteady lift can be re-constructed via the instantaneous vortex shedding frequency.

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